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VARIABLE PROPERTY TURBULENT HEAT AND MOMENTUM TRANSFER FOR AIR IN A VERTICAL ROUNDED CORNER TRIANGULAR DUCT

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Abstract—Experimental data over a limited Reynolds number range are reported for local friction and heat-transfer coefficients with constant heat flux for turbulent air flow in an electrically heated vertical, rounded corner triangular tube at local wall to bulk temperature ratios from 1·1 to 2·1. The test section was in fact a rounded corner, three equal sided duct, with a ratio of the corner radius of curvature to hydraulic diameter of 0·15. The Inconel test section had a 100-diameter entry section and a 180-diameter heated section. Bulk Reynolds numbers ranged from 10900 to 37000 at the inlet. Correlation in terms of the wall-modified Reynolds number is presented for the local friction factors. Correlations are presented for the local heat-transfer coefficients in terms of the wall-to-bulk temperature ratio with bulk properties; and also directly in terms of the wall-modified Reynolds number. Both the friction and heat-transfer results are shown to be significantly affected by the influence of variable properties. A noticeable deviation of about 10–15 per cent from similar correlations for circular tubes is noted in both the friction and heat-transfer results.

NOMENCLATURE

A_{flow}, flow area;
b, test section wall thickness;
c_p, specific heat;
D_h, hydraulic diameter, 4(flow area)/wetted perimeter;
f, fanning friction coefficient;
G, mass flux [lb/ft^2 h];
g_c, unit conversion 32·2 [lbft/lbf s^2];
H, enthalpy;
h, heat-transfer coefficient [Btu/h ft^2 °F];
J, unit conversion 778 [ft lbf/Btu];
k, thermal conductivity [Btu/h ft °F];
L, heated length of test section;
$\mathring{m}$, mass flow rate [lb/h];
Nu, Nusselt number, hD_h/k;
p, pressure;
Per, wetted perimeter;
Pr, Prandtl number, $\mu c_p/k$;
q', linear heat flux [Btu/h ft];
r, peripheral distance, corner to midwall;
$\bar{R}$, gas constant;
R, recovery factor;
Re, Reynolds number;
T, temperature;
V, bulk mean velocity;
x, distance down test section from thermal entry.

Greek symbols
ϵ, root mean square roughness;
γ, ratio of specific heats;
ρ, density [lb/ft^3];
Φ, impulse function, see equation (4);
τ, shear stress;
μ, viscosity.

Subscripts
aw, adiabatic wall;
e, expanded conditions;

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f, film;
iso, isothermal conditions;
0, stagnation conditions;
w, wall.

Note: The absence of a subscript on dimensionless parameters and gas properties denotes bulk static properties.

INTRODUCTION

IN MANY present day applications, such as nuclear reactors, high performance heat exchangers, or rocket power plants, design considerations dictate the use of heat-transfer ducts with noncircular cross sections. Because of the critical design limitations in many of these applications, the effects of variable properties on the heat-transfer and friction coefficients is a subject of current interest. It was the purpose of this research project to determine the effects of variable gas properties on the heat-transfer and friction coefficients for the turbulent flow of air through a duct of nominally equilateral triangular cross-section.

A review of previous works with triangular ducts for constant properties indicated that little deviation from circular tube correlations for heat-transfer and friction coefficients was to be expected for the equilateral triangular cross section [1]. However, significant deviations from circular tube correlations could be predicted for sharp cornered ducts such as a narrow apex angle isosceles triangle [2], or where the peripheral variations in duct surface temperature were significant [3]. Lowdermilk *et al.* [4] obtained measurements for air flow in an equilateral triangular duct of $L/D_h = 53$ with moderate property variations and found that their data for *average* values of the heat-transfer coefficient correlated well with their previous work in circular ducts when the properties were evaluated at mean film temperature and the hydraulic diameter was used. These experimenters found a significant variation in peripheral surface temperature (110 degF from corner to midwall for high heat fluxes and high flow rates), so that their correlation uses an average of the midwall and corner temperatures, for the wall temperature. Their average friction coefficient results also were correlated by the circular tube correlation using film properties with the hydraulic diameter.

Eckert and Irvine [2] found a large deviation from circular tube heat-transfer results in tests made with an 11·5° apex angle isosceles triangle. This deviation can be explained in terms of the flow conditions found in narrow angle noncircular ducts. Their tests were made under essentially constant property conditions and the local heat-transfer coefficient was based on the peripheral average wall temperature.

Carlson and Irvine [5] have run extensive tests for friction coefficients in triangular shaped ducts for fully developed adiabatic turbulent flow. Using the hydraulic diameter for correlating, they found an increasing deviation from the usual circular tube correlations as the apex angle became small. However, for the equilateral triangle their results are only 2 per cent below the Blasius correlation. On the other hand, for rectangular ducts, Hartnett *et al.* [6] found the circular tube correlation to hold very well using the hydraulic diameter. Their conditions were also adiabatic flow.

No work on the effects of variable properties on local heat-transfer coefficients in noncircular ducts is presently available in the literature. Previous works on these effects in circular ducts include those of Perkins and Wørsoe-Schmidt [7], McEligot and Magee [8], and Dalle Donne and Bowditch [9].

In the present experiment, the heat-transfer apparatus consisted of a vertical Inconel 600 tube with a 100-diameter hydrodynamic entry section and a 180-diameter heated section. Ideally, this gave a uniform inlet temperature and a step change in heat flux at the wall. Local friction coefficients were determined from pressure drop information at five pressure taps located at x/D's from 0 to 101. Eleven chromel–alumel thermocouples were spaced midwall along the heated length of the tube and were used to measure the local wall temperature variations from which the local heat-transfer coefficients could be determined.

Table 1 summarizes the range of variables covered in the present work.

Table 1. Range of variables in the present experiment

Experimental runs	19
Inlet bulk Reynolds number	10900–37000
Exit bulk Reynolds number	5800–24000
Maximum T_w/T_b	2·11
Axial distances	
Heat-transfer coefficients	6·0–157·0
Friction factor	14·5–72·0
Maximum Mach number	
Heat-transfer coefficients	0·33
Friction factor	0·22
Hydraulic diameter	0·0692 in
Corner radius of curvature/hydraulic diameter	0·15

APPARATUS AND TEST PROCEDURE

The experimental apparatus is similar to that used on other experiments of this type [7, 8]. Briefly, it consisted of a filtered compressed air supply, a pressure regulator valve, a Brooks rotameter for measuring flow rate, a Bourdon tube pressure gage to measure pressures at the flow meter and the first pressure tap, five water and/or mercury U-tube manometers to measure pressure drops, an inlet mixer with thermocouple probe where inlet stagnation temperature was measured, the test section, a water cooling jacket on the exhaust end of the test section, and a micro-valve for controlling the flow rate. The electrical apparatus used in heating the test section consisted of a Sorenson voltage regulator to stabilize the input voltage from a standard 110-V, 60-c a.c. source, a power transformer, a Variac potentiometer to control output voltage, a semi-fixed current transformer, quarter per cent voltmeter and ammeter to determine input power, and copper cables to connect the electrical heating system to the test section electrodes.

The test section consisted of an extruded Inconel 600 tube, typical of that commercially available, with the *nominal* cross-sectional shape of an equilateral triangle. Its nominal size was 0·125-in outside wall length with 0·015-in wall thickness. Actual dimensions of the tube were determined upon completion of the experimental work using photographs of approximately 25 × scale taken with a Ricchert metallograph. Photographs were taken of cross-sections cut from the test section at pressure taps 2 and 4. The dimensions at these two points, which differed by less than 1 per cent, were averaged and used in the later calculations. Thus, the hydraulic diameter used in the correlation in this paper is based on the photographically determined flow area and inside wall perimeter. The effects of corner curvature are therefore included in D_h. The corner radius of curvature, see Fig. 1, was determined to lie between 0·010 and 0·0115 in. The ratio of the corner radius of curvature to hydraulic diameter is approximately 0·15. It was necessary to obtain these accurate dimensions since significant variation from expected results for the adiabatic friction factor occurred using nominal dimensions. Since the characteristic dimension, D_h, appears to the fifth-power in the equation for determination of the friction factor, a small variation in dimensions was considerably magnified in the results of this equation. A photograph of the cross-sectional shape is shown in Fig. 1.

A qualitative estimate of the tube roughness was also made by examining the inner tube surface under 400 × magnification upon completion of the experiment. Although the inner tube wall exhibited little actual roughness, except in the corners where thermal stress cracks were evident, on a relative basis, the parameter ϵ/D_h was estimated to be 0·001.

The test section itself was 24-in long, with a 7-in entry section, 12-in heated section and a 5-in exit section. A stainless steel electrode was welded to the lower end of the heated section, as was a copper electrode to the upper end. Five $\frac{1}{16}$-in O.D., 6-mil wall thickness Inconel pressure taps were attached to the test section using Nioro braze. Five-mil holes were "drilled" through the tube walls after attachment of the pressure taps using an Elox electrical discharge machine which leaves no burr. Thus, the ratio of the hole diameter to test section diameter was less than 1:10.

Sixteen chromel–alumel thermocouples were spot-welded to the tube, eleven in the heated section, to record outside wall temperature variations along the tube. A ceramic and a glass thermocouple probe were both used to determine corrections for the "fin effect" at each thermocouple location. An additional thermocouple was attached at the flow meter to determine the air flow temperature there. A Minneapolis–Honeywell self-balancing indicating potentiometer was used to determine thermocouple e.m.f.'s.

A heat shield completely enclosing the test section was used to confine the convective air currents about the tube and provide a measure of stability to the external heat losses. Calibration runs over a range of heat fluxes were made to determine the external heat loss at each thermocouple as a function of local temperature difference, $T_w - T_{box}$.

DEFINITIONS AND CALCULATIONS

The local heat-transfer coefficient, h, is defined by the equation

$$q'_w = h\, Per_e\,(T_w - T_{aw}). \tag{1}$$

The term local is used here to mean a heat-transfer coefficient at a particular x location on the tube length. Since only a midwall thermocouple reading was used the local coefficient is based on this single temperature measurement and not on a peripherally averaged wall temperature. The variation of wall temperature around the periphery is small for this test section as will be discussed. The adiabatic wall temperature is defined by the relationship $T_{aw} = T + RV^2/2g_cJc_p$, where R is the recovery factor for turbulent flow in a tube, taken to be $Pr^{\frac{1}{3}}$ [8]. From an energy balance on an element of the tube, the heat flux to the air flow may be determined. The wall heat flux is determined by subtracting the radial and axial heat losses from the total electrical generation. The radial heat losses, which were generally less than 10 per cent of the total heat generated, are determined from the results of calibration runs made under no-flow conditions. The inside wall temperature is determined from the outside wall temperature measurement and the heat flux.

The bulk static temperature, T, is defined by analogy with the energy equation for constant properties as

$$c_{p,0}(T_0 - T) = V^2/2g_cJ \tag{2}$$

where $V = G_e\bar{R}T/P$ and $G_e = \mathring{m}/A_{\text{flow},e}$. The bulk stagnation temperature, T_0, can be found at any thermocouple location from an integration of the energy input to the gas. The local (fanning) friction factor is defined as

$$f = \tau_w/(\rho V^2/2g_c). \tag{3}$$

Momentum considerations for the steady flow of fluids through a duct lead to the following expression for the local wall shear stress in terms of quantities which may be determined experimentally:

$$\tau_w = -\frac{D_h}{4}\frac{\mathrm{d}}{\mathrm{d}x}\left[P + \rho\frac{V^2}{g_c}\right] = -\frac{D_h}{4}\frac{\mathrm{d}\Phi}{\mathrm{d}x}. \tag{4}$$

This expression assumes that the static pressure is constant across the flow and treats the momentum flux as one-dimensional.

The data reduction was done on the IBM 7072 at the University of Arizona Numerical Analysis Laboratory. Tables of gas properties extracted from [10] were loaded as input data. In preliminary calculations the program corrects for axial and radial thermal expansion at each thermocouple and pressure tap location, and fin effects at each thermocouple. Heat-transfer results are then obtained at the wall thermocouple locations. The axial conduction is found by using a subroutine to fit a parabolic curve through a plot of three adjacent thermocouple readings, and then determining the second derivative of temperature with respect to axial distance at the mid-thermocouple.

Local friction factors are determined at the

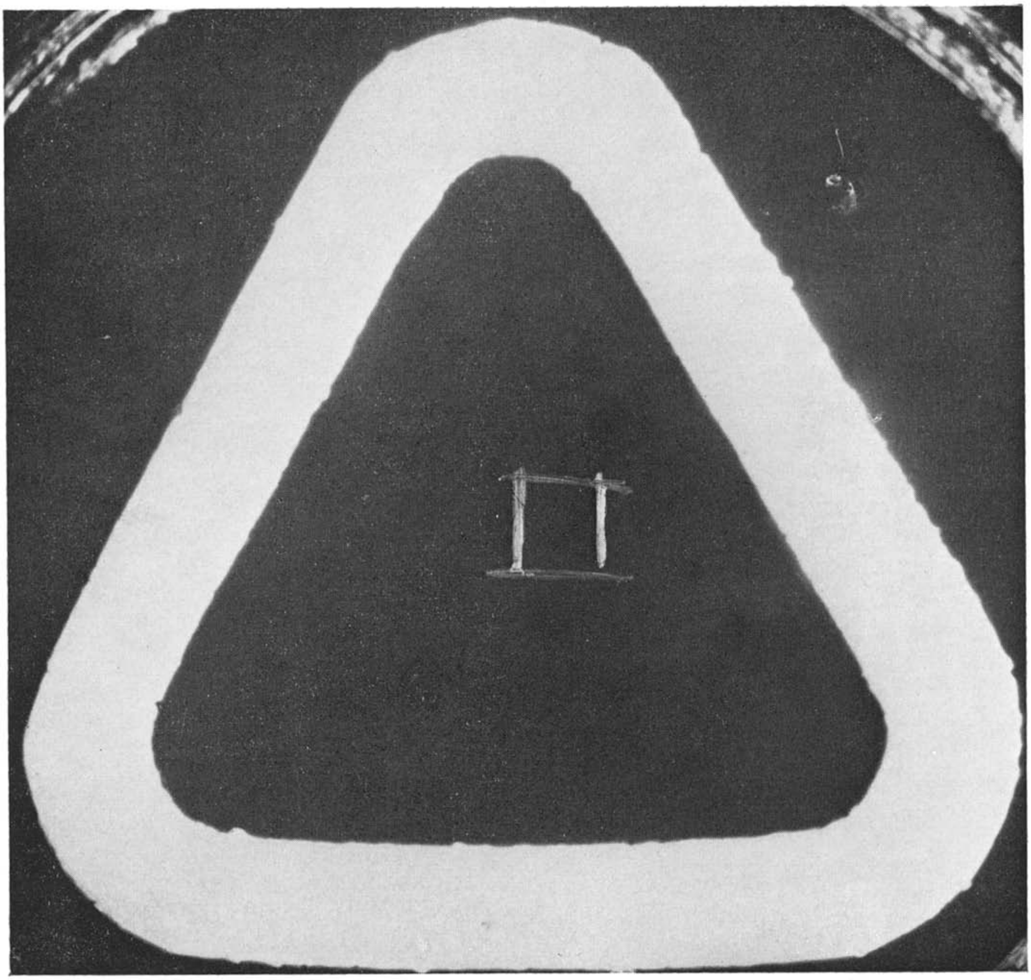

FIG. 1. Photograph of tube cross-section. $D_h = 0{\cdot}0692$ in, $A_{flow} = 0{\cdot}00478$ in^2, wall thickness $= 0{\cdot}015$ in.

pressure tap locations by interpolating the bulk static temperature to form the specific impulse function, Φ, then taking its gradient to obtain the wall shear stress.

No correction was made for peripheral temperature variation around the wall. Previous unpublished tests on a tube of square cross section with the same wall thickness and same side length had shown the midwall and corner temperatures to differ by at most 15 degF. This magnitude occurred only under the maximum heating conditions when the difference between the midwall and bulk temperatures was at least 500 degF.

If the analytical results of Deissler and Taylor [3] for peripheral wall temperature are used to predict the difference between the midwall and corner temperature it is found that at a heat flux of 50000 Btu/h ft^2, typical of these tests, the temperature difference is 16 degF. The peripheral wall temperature variations of Lowdermilk *et al.* [4] were some 12–60 per cent *lower* than predicted by Deissler and Taylor. Consequently it appears that the maximum peripheral wall temperature variation in this experiment was on the order of 15 degF. The midwall to bulk temperature difference under these maximum heating conditions was greater than 450 degF so that the peripheral effects can be neglected with confidence.

For the present nominally triangular test section the wall conduction parameter $k_w b/kD_h$ suggested in [2], was about 120. In contrast this parameter had a value of 24 in the tests of [2] and about 35 in the tests of [4].

RESULTS AND CORRELATIONS

One method of accounting for the appreciable variation of properties across the flow is to use a property or temperature ratio in the correlation which otherwise uses bulk properties. Another method uses a reference temperature such as the wall temperature.

The wall Reynolds number used in the correlations in this paper is defined as

$$Re_w = \frac{D_h \mathring{m}}{A\mu_w} \frac{T_b}{T_w} \tag{5}$$

and for a perfect gas becomes equivalent to

$$Re_w = \frac{V_b \rho_b D_h}{\mu_w}\left(\frac{\rho_w}{\rho_b}\right) = \frac{V_b \rho_w D_h}{\mu_w}.$$

This is usually called the modified wall Reynolds number.

Friction results

A series of runs was made to determine adiabatic friction factors for comparison with correlations of other researchers. These runs were interspersed with the heated runs to account for any minor changes in the tube characteristics, such as a gradual roughening due to oxidation or distortions in cross sectional shape due to thermal stresses. The data for these runs were reduced using a separate computer program that calculated the friction factor in the manner of Shapiro [11]. These results are shown in Fig. 2. All data show good agreement with the von Kármán–Nikuradse line. There is evidence of roughness effects ($\epsilon/D = 0{\cdot}001$) at the downstream taps at the higher Reynolds numbers. Taps 1–2 gave consistently low results, 5 per cent below the von Kármán–Nikuradse line, while the other taps tend to be a few per cent above the line.

As a result of using the parabolic curve fit procedure for the heated friction results data were obtained for taps 2, 3 and 4 only. Figure 3 shows the result of a correlation of the friction

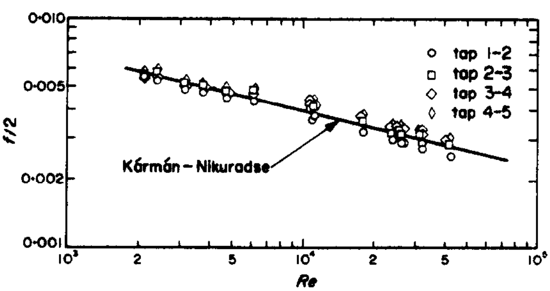

FIG. 2. Tap to tap isothermal friction coefficients for flow in a triangular duct.

data based on bulk properties but normalized by the present experimental isothermal friction factor evaluated at the same modified *wall Reynolds number*. This type of correlation is suggested by Perkins and Worsøe-Schmidt [7] and is derived from the works of Kutateladze

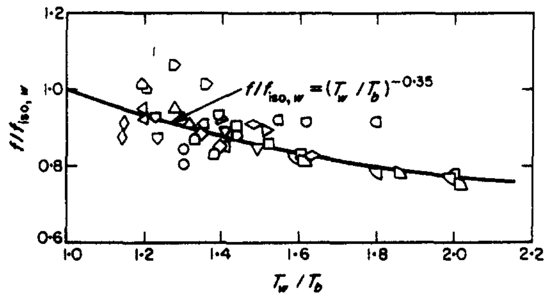

FIG. 3. Local friction coefficients based on bulk properties, normalized on the isothermal friction coefficients for the same wall Reynolds number.

and Leont'ev [12]. They derived the expression below for the *limiting* case of subsonic flow at *high Reynolds numbers* ($Re \to \infty$) with the isothermal friction coefficient based on the bulk Reynolds numbers:

$$f/f_{iso} = \left[\frac{2}{(T_w/T_b)^{\frac{1}{2}} + 1}\right]^2 \quad (6)$$

which may be reduced, in approximate form, to

$$f/f_{iso} = (T_w/T_b)^{-0.6}. \quad (7)$$

While Perkins and Worsøe-Schmidt found good agreement with the above relations for their data using the modified wall Reynolds number to evaluate the isothermal friction coefficient, the present results are better correlated by the relation

$$f/f_{iso,\,wall} = (T_w/T_b)^{-0.35}. \quad (8)$$

Most of the data lie within ± 10 per cent of this correlation while a few points are off by 15 per cent. The data shown in Fig. 3 were for taps 2, 3 and 4, located 14·5, 47 and 72 diameters downstream, respectively. The data for the first two taps are corrected for the effects of the thermal entrance region. These corrections were determined in the manner of [7].

The algebraic expression resulting from a series of plots to determine this entrance effect gives:

$$\frac{f_b}{f_{downstream}} = \left[\frac{T_w}{T_b}\right]^{\left(\frac{D_h}{x}\right)^{0.67}}. \quad (9)$$

This expression relates the increase in local friction coefficient in the thermal entry to a downstream value at the same T_w/T_b. By combining the downstream and entry correlations a single expression can be determined for the local friction coefficient as

$$\frac{f_b}{f_{iso,\,wall}} = \left[\frac{T_w}{T_b}\right]^{-0.40 + \left(\frac{D_h}{x}\right)^{0.67}}. \quad (10)$$

This correlation can be used to find the local friction coefficient at particular x/D and T_w/T_b values normalized on the isothermal friction value found using the modified wall Reynolds number. Since good agreement was found between the adiabatic data determined on the rounded corner triangular duct and the standard circular tube correlations, either the Blasius or von Kármán–Nikuradse correlations may be used to determine $f_{iso,\,wall}$. The correlation was determined for $14.5 < x/D_h < 72$ and $1.1 < T_w/T_b < 2.11$. However, it should be extendable over a wider x/D and T_w/T_b range; for example, x/D's from 5 to 200.

The fact that correlation was achieved with a lower exponent on the T_w/T_b parameter than for the circular tube would indicate that there is a significant difference between heated friction data for circular tubes and equilateral triangular tubes under similar conditions. If one considers equations (7) and (8), one sees that the present results obtained on the rounded corner triangular duct will be approximately 20 per cent higher than the corresponding circular value for a T_w/T_b of 2·0.

In an attempt to verify this result, the sharp cornered equilateral triangle data for average friction coefficients from Lowdermilk *et al.* [4]

were examined. Their correlation was based on film properties in both the friction factor definition and in the Reynolds number (the modified film Reynolds number). When one converts the film friction factor to a bulk definition and the film Reynolds number to a wall definition, to correspond to the correlation method used in this paper, the results show a dependence on the wall to bulk temperature ratio to the $-0{\cdot}75$ power. This is significantly higher than the results found in the present experiment, equation (8). A re-correlation of this data was attempted as is shown in Fig. 4. Their friction results are better correlated when multiplied by $(T_w/T_b)^{-0\cdot 3}$.

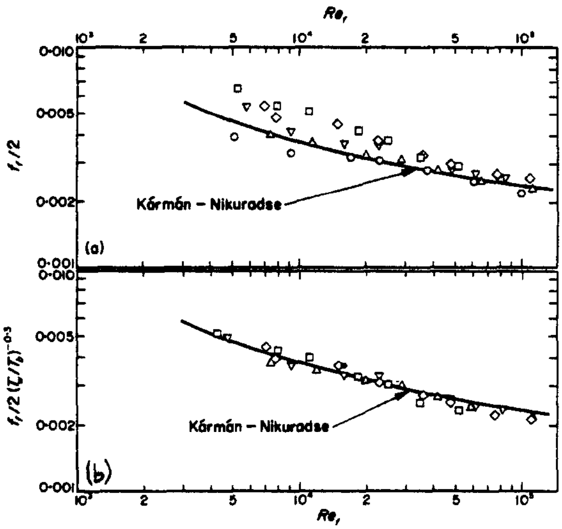

FIG. 4. Average friction coefficients of Lowdermilk *et al.* [4]: (a) directly plotted from their results; (b) as re-correlated.

This is the same as *reducing* the dependence on the wall to bulk temperature ratio to the $-0{\cdot}45$ power which is much closer to the present result. At a temperature ratio of two their re-correlated average results are some 8 per cent lower than the present results.

Heat-transfer results

Since bulk correlations are easier to use for design purposes, a correlation based upon bulk properties was tried.

It was necessary to find a suitable entry correction parameter which would correlate all the data from the first thermocouple at an x/D of 6·0 to the last usable thermocouple near the end of the test section, number 10, at an x/D of 123. Since the investigators of [7] had found a correction of the form $[1 + (x/D_h)^{-0\cdot7}(T_w/T_b)^{0\cdot7}]$ to be satisfactory, this correction was used. Figure 5 shows the results, with the data correlating to within ± 15 per cent, with all but a few points within ± 10 per cent. The resulting correlation is given by:

$$Nu = 0{\cdot}021\, Re^{0\cdot8}\, Pr^{0\cdot4}\, (T_w/T_b)^{-0\cdot7} \left[1 + (x/D_h)^{-0\cdot7}(T_w/T_b)^{0\cdot7}\right]. \qquad (11)$$

The entry correction was applied through and including an x/D of fifty. When compared with the results of investigators who studied the effects of variable properties in circular ducts, one notes that the correlation is about 14 per cent lower than that of [7]. Moreover, in the downstream region at a median T_w/T_b of 1·5, the results of equation (11) would be 10 per cent lower than the correlation of [8], where the exponent used on the temperature ratio term was $-0{\cdot}5$ instead of $-0{\cdot}7$, and the coefficient, 0·021, was identical to equation (11). It thus appears that the heat-transfer results are about 10 per cent lower than corresponding circular tube results, but the T_w/T_b effect is about the same.

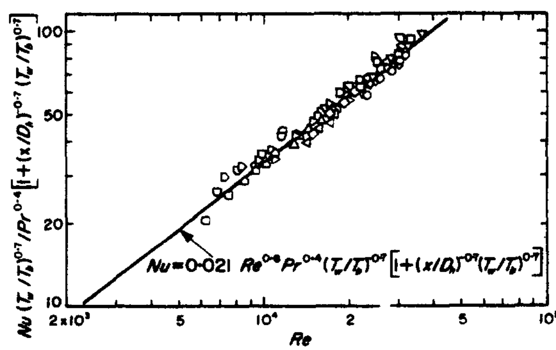

FIG. 5. Local heat-transfer results based on bulk properties. $6 < x/D_h < 125$; $1{\cdot}1 < T_w/T_b < 2{\cdot}11$

Next, a correlation in the manner of Perkins and Worsøe-Schmidt [7], based upon properties evaluated at wall temperatures and the wall modified Reynolds number, was tried. An entry correction identical to that used in the previous method of correlation was used in the upstream region for x/D's less than fifty. The results are shown in Fig. 6, where the correlation is given by:

$$Nu_w = 0{\cdot}0195\, Re_w^{0{\cdot}8}\, Pr_w^{0{\cdot}4} \left[1 + (x/D_h)^{-0{\cdot}7} (T_w/T_b)^{0{\cdot}7}\right]. \quad (12)$$

All the data is seen to fall within ± 8 per cent of the reference line given by equation (12).

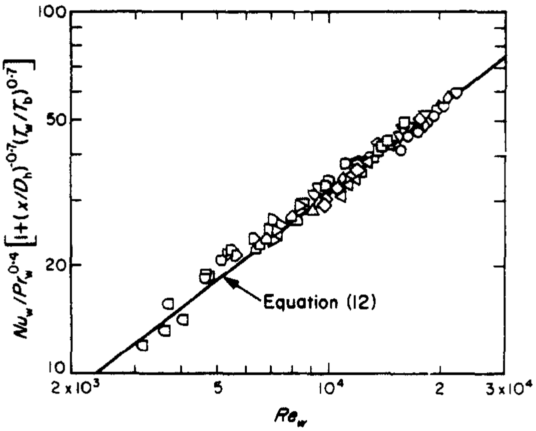

FIG. 6. Local heat-transfer results based on wall properties. $6 < x/D_h < 125$; $1{\cdot}1 < T_w/T_b < 2{\cdot}11$

The above result is about 15 per cent below that of [7] and 6 per cent below that of [8] for similar correlations based upon wall temperatures and wall modified Reynolds numbers. An examination of the data of Lowdermilk *et al.* [4] for a sharp cornered equilateral triangular duct reveals that the average Nusselt number based upon film temperature fell about 5–10 per cent below the correlation these same investigators found for circular ducts under similar conditions. Deissler and Taylor [3], in their constant property analytic approach to the heat-transfer parameters for equilateral triangles, predict that the average Nusselt number will be lower for the triangular duct than for the circular. They attribute the difference to the fact that the flow is nearly stagnant in the corners of the duct, hence the heat transfer approaches zero there. Their results are presented graphically for different values of their nonuniform peripheral temperature distribution parameter. For a value of this parameter similar to the one for the present tube Deissler and Taylor predict a Nusselt number for air that is about 15–20 per cent lower than circular tube data at $10000 < Re < 30000$. They plotted the results from [4] and found good agreement with their prediction, although the experimental data did not fall as far below the circular tube correlation as predicted. Deissler and Taylor suggest that this may be due to the fact that their analysis neglected the effects of secondary flow near the corners.

If one examines the photo of the present duct shown in Fig. 1, one observes that the corners are rounded, as opposed to the sharp corners that were considered in the triangular ducts of [3, 4]. One would expect that the flow in the corners of this duct would not be as stagnant as that in sharp-corner ducts. Therefore, the heat transfer, while lower in the corner than at the midwall, should not approach zero. Thus, one would not expect as great a peripheral temperature variation, nor as much divergence below circular tube data for the present tube as that predicted by [4]. Such is indeed the case.

CONCLUSIONS

From the results of this investigation of local heat-transfer and friction coefficients for air flow through a nominally equilateral triangular duct at wall temperatures to 1700°F, corresponding wall to bulk temperature ratios to 2:1 and bulk Reynolds numbers to 37000, it may be concluded:

1. The effect of variable properties upon the friction coefficient is less than that found by previous investigators for circular ducts, as evidenced by a lesser exponent on the T_w/T_b term in the friction correlation.

2. The local heat-transfer results in an equilateral triangular duct are about 10 per cent lower than circular duct results. Evidence from the experiment of Lowdermilk *et al.* [4] and the analysis of Deissler and Taylor [3] corroborates this conclusion.
3. The effect of variable properties upon the local heat-transfer coefficient is nearly identical with that found for circular ducts, as evidenced by both the correlations based upon bulk and wall properties.

The recommended correlations for the equilateral triangle data are:

Friction coefficients

$$f/f_{iso,w} = (T_w/T_b)^{-0.40 + \left(\frac{D_h}{x}\right)^{0.67}}$$

for $14.5 < x/D_h < 72$; $1.10 < T_w/T_b < 2.11$.

Heat-transfer coefficients
Bulk correlation.

$$Nu = 0.021\, Re^{0.8}\, Pr^{0.4} (T_w/T_b)^{-0.7} [1 + (x/D_h)^{-0.7} (T_w/T_b)^{0.7}]$$

for $1.10 < T_w/T_b < 2.11$; $6.0 < x/D_h < 123$,

with the same correlation without the entry parameter for $x/D_h > 50$.

Wall correlation.

$$Nu_w = 0.0195\, Re_w^{0.8}\, Pr_w^{0.4} [1 + (x/D_h)^{-0.7} (T_w/T_b)^{0.7}]$$

for $1.1 < T_w/T_b < 2.11$; $6.0 < x/D_h < 123$,

with the same correlation without the entry parameter for $x/D_h > 50$.

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REFERENCES

1. W. M. Kays, *Convective Heat and Mass Transfer.* McGraw-Hill, New York (1966).
2. E. R. G. Eckert and T. F. Irvine, Pressure drop and heat transfer in a duct with triangular cross-section, *J. Heat Transfer* **82**, 125–137 (1960).
3. R. G. Deissler and M. F. Taylor, Analysis of turbulent flow and heat transfer in non-circular passages, NASA TR R-31 (1959).
4. W. H. Lowdermilk, W. F. Wieland and J. N. B. Livingwood, Measurement of heat transfer and friction coefficients for flow of air in non-circular ducts at high surface temperatures, NACA RM E53J07 (1954).
5. L. W. Carlson and T. F. Irvine, Fully developed pressure drop in triangular shaped ducts, *J. Heat Transfer* **83**, 441–444 (1961).
6. J. P. Hartnett, J. C. Y. Koh and S. T. McComas, A comparison of predicted and measured friction factors for turbulent flow through rectangular ducts, *J. Heat Transfer* **84**, 82–88 (1962).
7. H. C. Perkins and P. Worsøe-Schmidt, Turbulent heat and momentum transfer for gases in a circular tube at wall to bulk temperature ratios to seven, *Int. J. Heat Mass Transfer* **8**, 1011–1031 (1965).
8. D. M. McEligot and P. M. Magee, Aerothermodynamic studies at high temperature, TR 326-1, Mechancial Engineering Dept., Stanford University (1962).
9. M. Dalle Donne and F. H. Bowditch, Local heat transfer and average friction coefficients for subsonic laminar, transitional, and turbulent flow of air in a tube at high temperatures, *Nucl. Engng, Lond.* **8**, 20–29 (1963).
10. J. C. Hilsenrath, W. Beckett, W. S. Benedict, L. Fano, H. J. Hoge, J. F. Masi, R. L. Nuttall, Y. S. Touloukian and H. W. Wooley, NBS Circular, 564 (1955).
11. A. H. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow,* Vol. 1. Ronald Press, New York (1953).
12. S. S. Kutateladze and A. I. Leont'ev, *Turbulent Boundary Layers in Compressible Gases,* Translated by D. B. Spalding. Arnold, London (1964).

Résumé—Des résultats expérimentaux sont donnés dans une gamme limitée de nombres de Reynolds pour les coefficients locaux de frottement et de chaleur avec un flux de chaleur constant pour un écoulement d'air turbulent dans un tube vertical à section triangulaire à angles arrondis et chauffé électriquement, avec un rapport de la température pariétale locale à la température globale allant de 1,1 à 2,1. La partie essayée était effectivement un tuyau dont la section droite était un triangle équilatéral à angles arrondis, avec un rapport du rayon de l'arrondi au diamètre hydraulique égal à 0,15. La partie essayée était en Inconel et avait une région d'entrée hydrodynamique longue de 100 diamètres et une région chauffée longue de 180 diamètres. Les nombres de Reynolds globaux variaient de 10900 à 37000 à l'entrée. On présente une corrélation pour les coefficients locaux de convection en fonction du nombre de Reynolds pariétal modifié, ainsi que pour les coefficients locaux de transport de chaleur en fonction du rapport des températures pariétale et globale avec les propriétés globales, et aussi directement en fonction du nombre de Reynolds pariétal modifié. On montre que les résultats de frottement et de transport de chaleur sont affectés sensiblement tous les deux par l'influence des propriétés variables. Une déviation importante d'environ 10 à 15% à partir de corrélations similaires pour des tubes circulaires est remarquée à la fois avec les résultats de frottement et de transport de chaleur..

Zusammenfassung—Für einen begrenzten Bereich von Reynolds-Zahlen werden experimentelle Daten angegeben für die lokalen Reibungsbeiwerte und Wärmeübergangszahlen bei konstanter Wärmestromdichte und turbulenter Luftströmung. Verhältnisse von örtlicher Wandtemperatur zu Gastemperatur lagen zwischen 1,1 und 2,1. Als Versuchsstrecke diente ein elektrisch beheizter Kanal mit dem Querschnitt eines gleichseitigen Dreiecks mit abgerundeten Ecken; wobei das Verhältnis des Radius der Eckenausrundung zum hydraulischen Durchmesser 0,15 betrug. Die Versuchsstrecke aus Inconel besass eine hydrodynamische Einlaufstrecke von 100 mal dem Durchmesser und eine beheizte Strecke von 180 mal dem Durchmesser. Die mit den Stoffwerten der Gasströmung gebildeten Reynolds-Zahlen lagen am Einlauf zwischen 10900 und 37000. Es wird eine Beziehung für die lokalen Reibungsbeiwerte geliefert in Abhängigkeit von der Reynolds-Zahl, errechnet mit den Stoffwerten an der Wand. Ebenso werden Beziehungen angegeben für die lokalen Wärmeübergangszahlen in Abhängigkeit vom Verhältnis der Wandtemperatur zur Temperatur des Gasstromes, mit den Stoffwerten des Gasstromes, und auch direkt in Abhängigkeit von der Reynolds-Zahl, die mit den Stoffwerten an der Wand gebildet wurden. Sowohl die Reibung als auch der Wärmeübergang zeigen einen deutlichen Einfluss der veränderlichen Stoffwerte. Eine merkliche Abweichung von 10 bis 15% gegenüber ähnlichen Beziehungen für Rohre mit Kreisquerschnitt wird bei den Ergebnissen für die Reibung und den Wärmeübergang festgestellt.

Аннотация—В ограниченном диапазоне изменения чисел Рейнольдса приводятся экспериментальные данные по локальным коэффициентам трения и теплообмена при постоянном тепловом потоке при турбулентном течении воздуха в электрически нагретом канале с сечением в виде треугольника с закругленными углами при локальном отношении температуры стенки к температуре ядра потока от 1,1 до 2,1. Рабочим участком, фактически, является канал с сечением в виде равностороннего треугольника с закругленными углами с отношением радиуса закругления к гидродинамическому радиусу, равном 0,15. Инконелевый рабочий участок имеет входную секцию длиной в 100 диаметров и нагреваемую секцию длиной в 180 диаметров. Числа Рейнольдса в ядре потока на входе изменялись от 10900 до 37000. Дано выражение для локального коэффициента трения, включающее число Рейнольдса, определенное из условий на стенке. Приводятся выражения для локальных коэффициентов теплообмена, включающее отношение температуры ядра потока к температуре стенки со свойствами, определенными по температуре ядра, а также в виде числа Рейнольдса, определенного по температуре стенки. Показано, что переменность свойств значительно влияет как на трение, так и на теплообмен. В данных по трению и теплообмену наблюдается заметное отклонение ~10–15% от аналогичных выражений для круглых труб.